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QUASI-SELF-SIMILAR DESCRIPTION OF AN EXHAUST JET

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The method of construction of an analytical solution for the far field of an exhaust jet on the basis of the quasi-self-similar solution of Prandtl equations and the turbulence model with one differential equation for the coefficient of kinematic viscosity is described. An exact numerical solution for distances to about 10^1 radii of the nozzle is constructed for the basic version. Then, a numerical solution and its analytical approximation by the quasi-self-similar solution are constructed. Approximations of the similarity parameters of the self-similar problem as functions of the similarity parameters of the initial problem in the form of polynomials allow construction of analytical solutions for different situations, which are in satisfactory agreement with the exact numerical solution at distances of ~10^3 radii.

The problem of description of a turbulent jet is related, first of all, to the selection of the turbulence model. There exist models of isotropic turbulence (Prandtl and Taylor) which involve only algebraic relations between the coefficient of viscosity and the velocity gradient [1] and models with one [2, 3], two (k–ε model [4]), and three differential equations and different versions of them [5]. We dwell on the model with one differential equation. The isobaric axisymmetric jet ($p \approx p_\infty$, $\rho = p_\infty m/RT$) is described by the following system:

$$\rho u \frac{\partial \nu}{\partial x} + \rho v \frac{\partial \nu}{\partial r} = \frac{2}{r}\frac{\partial}{\partial r}\left\{ r\mu \frac{\partial \nu}{\partial r} \right\} + 0.2\mu \left| \frac{\partial u}{\partial r} \right| , \quad \mu = \rho\nu , \tag{1}$$

$$\frac{\partial \rho u r}{\partial x} + \frac{\partial \rho v r}{\partial r} = 0 , \tag{2}$$

$$\frac{\partial \rho u^2}{\partial x} + \frac{1}{r}\frac{\partial \rho u v r}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\mu \frac{\partial u}{\partial r} \right\} , \tag{3}$$

$$\frac{\partial \rho u H}{\partial x} + \frac{1}{r}\frac{\partial \rho v r H}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left\{ \frac{r\mu}{\mathrm{Pr}} \frac{\partial H}{\partial r} \right\} + \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\mu \left(1 - \frac{1}{\mathrm{Pr}} \right) u \frac{\partial u}{\partial r} \right\} , \quad H = C_p T + \frac{u^2 + v^2}{2} , \tag{4}$$

$$\frac{\partial \rho u Y}{\partial x} + \frac{1}{r}\frac{\partial \rho v r Y}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left\{ \frac{r\mu}{\mathrm{Sc}} \frac{\partial Y}{\partial r} \right\} , \quad Y = \frac{\rho_v}{\rho} , \quad \mathrm{Sc} = \frac{\mu}{\rho D} , \quad \mathrm{Pr} = \frac{C_p \mu}{k} . \tag{5}$$

We write the boundary conditions. At $x = 0$

$$u = u_{\mathrm{a}} , \quad v = 0 , \quad H = H_{\mathrm{a}} \equiv C_p T_{\mathrm{a}} + u_{\mathrm{a}}^2/2 , \quad \nu = 0.001\nu_{\mathrm{a}} , \quad Y = Y_{\mathrm{a}} , \quad 0 \le r < r_{\mathrm{a}} ; \tag{6}$$

$$u = u_\infty , \quad v = 0 , \quad H = H_\infty \equiv C_p T_\infty + u_\infty^2/2 , \quad \nu = 0.001\nu_{\mathrm{a}} , \quad Y = Y_\infty , \quad r_{\mathrm{a}} \le r \le r_{\mathrm{m}} ; \tag{7}$$

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$$\nu(0, r_a) = \nu_a \equiv 0.014 r_a \, |u_a - u_\infty| , \quad r = r_a . \tag{8}$$

Here u_a and T_a are the velocity and temperature at the cut of the nozzle with a radius r_a. On the jet axis $r = 0$ and beyond the jet at $r = r_m$ we fulfill the conditions of symmetry and decay of disturbances within the entire field along the longitudinal coordinate $0 \le x \le x_m$:

$$\frac{\partial \nu}{\partial r} = 0 , \quad \frac{\partial u}{\partial r} = 0 , \quad v = 0 , \quad \frac{\partial H}{\partial r} = 0 , \quad \frac{\partial Y}{\partial r} = 0 . \tag{9}$$

At a large distance from the jet in the concurrent flow at $r = r_m$ we can also specify conditions (7) instead of (9).

We relate the density ρ, the velocity components u and v, and the enthalpy H to the parameters in the concurrent flow ρ_∞, u_∞, and H_∞, the coordinates x and r to r_a, and the coefficients of turbulent dynamic viscosity μ, thermal conductivity k, and diffusion D to the characteristic quantities $\mu_a = \rho_\infty \nu_a$, $k_a = C_p \mu_a/\mathrm{Pr}$, and $D_a = \nu_a/\mathrm{Sc}$. As the similarity parameters we obtain the concurrency $1/s = u_\infty / u_a$, the parameter of heating $h = H_a / H_\infty$, and the relative mass concentration of the vapor at the nozzle cut Y_a and in the atmosphere Y_∞. The Reynolds number $\mathrm{Re} = u_\infty r_a / \nu_a = 1/(0.014 |s - 1|)$ is expressed in terms of the concurrency. We assume the turbulent Prandtl Pr and Schmidt Sc numbers to be equal to unity, although, in the general case, the method under development holds for $\mathrm{Pr} \ne 1$ and $\mathrm{Sc} \ne 1$. The problem was solved numerically using the implicit finite-difference scheme [6]. At the distance $x \ge 10 r_a$, the initial portion ends and the excess velocity $u_1 = u - u_\infty$, the enthalpy $H_1 = H - H_\infty$ (temperature $T_1 = T - T_\infty$), and the vapor concentration $Y_1 = Y - Y_\infty$ sharply decrease within the intermediate range. The coefficient of kinematic viscosity first quickly increases to a certain cross section $x_{m\nu}$ and then slowly decreases. In the Prandtl model of turbulence, the distance is $x_{m\nu} \sim 10 r_a$. In the model with one differential equation, the distance $x_{m\nu}$ is several times larger and the coefficient of turbulent viscosity decreases much more slowly than in the Prandtl model. At the distance $x \sim 10^2 r_a$, the solution approaches the self-similar one. When $x \sim 10^3 r_a$, the complete self-similarity is also not observed.

We approximate the exact numerical solution by the analytical one using the quasi-self-similar solution. We write the initial equations (1)–(5) at large distances $x \ge 10^2 r_a$ in the coordinate system tied to the atmosphere:

$$\frac{\partial \nu_1}{\partial t} \approx \frac{2}{r} \frac{\partial}{\partial r} \left\{ r \nu_1 \frac{\partial \nu_1}{\partial r} \right\} + 0.2 \nu_1 \left| \frac{\partial u_1}{\partial r} \right| , \quad \frac{\partial u_1}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \nu_1 \frac{\partial u_1}{\partial r} \right\} , \tag{10}$$

$$\frac{\partial H_1}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \nu_1 \frac{\partial H_1}{\partial r} \right\} , \quad \frac{\partial Y_1}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \nu_1 \frac{\partial Y_1}{\partial r} \right\} , \quad \nu_1, u_1, H_1, Y_1 \big|_{r \to \infty} \to 0 . \tag{11}$$

We use the self-similar variables

$$\nu_1 (r, t) = t^n f(\eta) , \quad \eta = \frac{r^2}{8 t^{n+1}} , \quad u_1 = t^p w(\eta) , \quad p = \frac{n-1}{2} , \quad H_1 = t^m q(\eta) , \quad Y_1 = t^g z(\eta) . \tag{12}$$

The system is reduced to the ordinary differential equations

$$nf - (n+1) f' \eta = (\eta f f')' + 0.2 f \sqrt{\eta/2} \, |w'| , \quad pw - (n+1) w' \eta = (\eta f w')' / 2 ,$$

$$mq - (n+1) q' \eta = (\eta f q')' / 2 , \quad gz - (n+1) z' \eta = (\eta f z')' / 2 . \tag{13}$$

Disclosing the singularity of system (10)–(11) on the axis at $r = 0$, we find the conditions for the derivatives at $\eta = 0$:

$$f'(0) = n ; \quad w'(0) = \frac{2 p w_0}{f_0} , \quad f_0 = \frac{\nu_{10}}{t_{10}^n} , \quad w_0 = \frac{u_{10}}{t_{10}^p} , \quad q'(0) = \frac{2 m q_0}{f_0} , \quad q_0 = \frac{H_{10}}{t_{10}^m} ; \quad z'(0) = \frac{2 g z_0}{f_0} , \quad z_0 = \frac{Y_{10}}{t_{10}^g} .$$

Here ν_{10}, u_{10}, H_{10}, and Y_{10} are the numerically found values of the sought quantities in a certain cross section $x = x_{10} = u_\infty t_{10}$, beginning with which we will construct the quasi-self-similar approximation. At the edge of the jet, at $\eta = \eta_m$ the sought functions w, q, and $z(\eta_m)$ must simultaneously take zero values and the function f must take a low nonzero value. We relate the functions sought to the known values at zero $F = f/f_0$, $W = w/w_0$, $Q = q/q_0$, and $Z = z/z_0$ and the independent variable to the maximum value $\xi = \eta/\eta_m$. From (13) we obtain

$$nF - (n+1)\,F'\xi = A\,(\xi FF')' + BF\sqrt{\xi}\,\left|W'\right|\,, \quad pW - (n+1)\,W'\xi = \frac{A}{2}\,(\xi FW')'\,,$$

$$mQ - (n+1)\,Q'\xi = \frac{A}{2}\,(\xi FQ')'\,, \quad gZ - (n+1)\,Z'\xi = \frac{A}{2}\,(\xi FZ')'\,, \quad A = \frac{f_0}{\eta_m}\,, \quad B = \frac{0.2w_0}{\sqrt{2\eta_m}}\,, \tag{14}$$

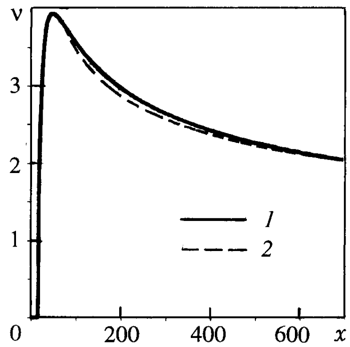
$$F'(0) = \frac{n}{A}\,, \quad W'(0) = \frac{2p}{A}\,, \quad Q'(0) = \frac{2m}{A}\,, \quad Z'(0) = \frac{2g}{A}\,; \quad F(0) = 1 = W(0) = Q(0) = Z(0)\,.$$

It is evident that the condition of simultaneous vanishing of the three sought functions at $\xi = 1$ requires the equality $g = m = p$. This is confirmed by the numerical solution. Self-similarity is violated, since $p \neq (n-1)/2$. The similarity parameter B in (14) must be multiplied by the quantity $t_{10}^{p-(n-1)/2}$, which is close to unity: $B = 0.2w_0 t_{10}^{p-(n-1)/2}/\sqrt{(2\eta_m)} = 0.4u_{10}t_{10}/r_{10}$, where $r_{10} = \sqrt{(8\eta_m t_{10}^{n+1})}$ is the transverse dimension of the jet. The problem is reduced to construction of the solution for the first two ordinary differential equations of second order from system (14) with unit values of the sought functions assigned at zero and with the derivatives involving free parameters n/A and $2p/A$. We find the sought values of the exponents n and p from the exact numerical solution on the jet axis: $n \approx \ln(\nu_{10}/\nu_e)/\ln(x_{10}/x_e)$ and $p \approx \ln(u_{10}/u_e)/\ln(x_{10}/x_e)$. The values of the quantity $\eta_{m0} = r_{10}^2/(8t_0^{n+1})$ which correspond to the self-similar solution give a shift of the time origin in the coordinate system tied to the atmosphere by the value $\Delta t = t_{10} - t_0$ (or $\Delta x = x_{10} - x_0$ in the system tied to the airplane nozzle), where t_0 is such that the verified similarity parameters $A = f_0/\eta_{m0} = (t_0/t_{10})^n \nu_{10} t_0 8/r_{10}^2$ and $B = 0.2w_0 t_{10}^{p-(n-1)/2}/\sqrt{(2\eta_{m0})} = 0.4u_{10}t_0/r_{10}(t_0/t_{10})^p$ simultaneously with the above-given exponents n and p allow fulfillment of the zero conditions of the sought functions $W = Q = Z$ at the jet edge at $\xi = 1$. In the self-similar solution at very large distances $x > 10^4 r_a$, the derivative of the function $F(\xi)$ has a singularity when $\xi \to 1$: $F'(\xi) \sim 1/(1-\xi)^{1/2}$. In the quasi-self-similar solution at closer distances $x \sim (10^2\text{–}10^3)r_a$, it is sufficient to use, instead of $F(\xi = 1) = 0$, a softer condition of a decrease of an order of magnitude in the function $F(\xi = 1)$ compared to its maximum value at zero $F(\xi = 0) = 1$.

We consider two versions of approximation of the exact solution by the quasi-self-similar one with an example of the exhaust jet from an Il-96 airplane (engine PS-90A) in cruising flight mode at a height of 11 km: $r_a = 0.712$ m, $p_\infty = 22{,}690$ N/m^2, $u_\infty = 236$ m/sec, $u_a = 404$ m/sec, $T_\infty = 216.7$ K, $T_a = 287$ K, $s = 1.712$, $h = 1.506$, $Y_a = 0.0039$, and concentration of a saturated vapor above ice $Y_{s.i}(T_\infty) = Y_{s.i\infty} = 4.58\cdot10^{-5}$. The last quantity is necessary for calculation of the concentration of the condensate (ice) in the aerosol:

$$Y_i = Y - Y_{s.i}\,(T) = Y - Y_{s.i\infty}\exp\left\{\int_{T_\infty}^{T}\frac{m_w L dT}{RT^2}\right\}. \tag{15}$$

1. We approximate the exact solution by the self-similar one within the range $x = (10^2\text{–}10^3)r_a$. For the above-given *basic* version, the exact numerical solution in the initial cross section $x_{10} = 71.2$ m gives the following values: $\nu_{10} = 3.80$ m^2/sec, $u_{10} = 36.6$ m/sec, $H_{10} = 2.69\cdot10^4$ J/kg ($T_{10} = 17.5$ K), $Y_{10} = 8.49\cdot10^{-4}$, jet radius $r_j = 2.84$ m, and condensation-trail radius $r_c = 2.58$ m. In the end cross section $x_e = 712$ m, we obtained $\nu_{1e} = 2.02$ m^2/sec, $u_{1e} = 4.98$ m/sec, $H_{1e} = 4700$ J/kg ($T_{1e} = 2.45$ K), $Y_{1e} = 1.16\cdot10^{-4}$, $r_{j.e} = 7.72$ m, and $r_{c.e} = 4.59$ m. For quasi-self-similar problem (14) we calculated $A = 1.338$, $B = 1.603$, $n = -0.2745$, and $p = m = g = -0.8663$. The solution is well approximated by the polynomials of the second degree



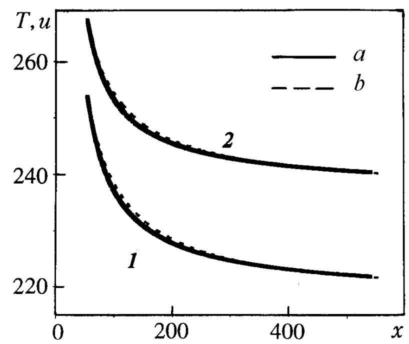


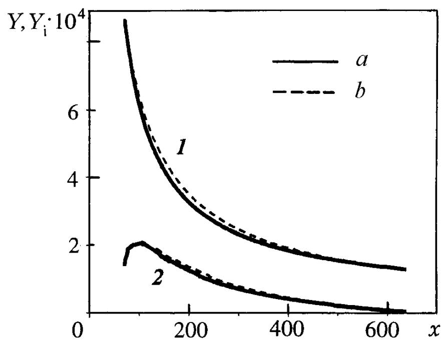
Fig. 1. Coefficient of kinematic viscosity ν along the axis of the jet behind the Il-96 airplane in the cruising-flight mode (base): 1) numerical solution; 2) quasi-self-similar solution constructed within the range $\Delta x = (10^2–10^3)r_a$. ν, m^2/sec; x, m.

Fig. 2. Temperature T (1) and velocity u (2) on the axis of the jet behind the Il-86 airplane in the cruising-flight mode: a) numerical solution; b) quasi-self-similar solution constructed on the basis of the approximation of the solution for Il-96. T, K; u, m/sec; x, m.

$$F = 0.9966 - 0.3733\xi - 0.5256\xi^2 , \quad W = Q = Z = 0.9908 - 1.1858\xi + 0.2101\xi^2 . \tag{16}$$

Figure 1 gives the exact solution $\nu(x, r = 0)$ and its approximation. Quasi-self-similar approximation gives a slight overstatement of the values for the profiles of the quantities $\nu(r)$, $u(r)$, $T(r)$, and $Y(r)$ at the edge of the jet in the initial cross section x_{10} and narrows the jet in the end cross section x_e by the value $\Delta r/r_{j.e} \le 0.1$. Good agreement has also been obtained for the concentration of the condensate Y_i, which, in view of the exponential relation (15), is very sensitive to weak changes in T and Y. This quantity can serve as an approximation criterion: with satisfactory approximation of the quantity Y_i the remaining sought functions, first of all T and Y, will also be in good agreement with the exact solution. The quantities A, B, n, and p and the functions F and W do not depend on Y_a and slightly change within the range s = 1.6–1.8, h = 1.5–2.0 of the similarity parameters of the initial problem (1)–(5). Comparison with the exact solution showed that slight changes in the solution (16) can be neglected. The dependences of the similarity parameters A, B, n, and p on s and h are close to linear ones and are easily approximated for other conditions of flight and other airplanes. For example, for an Il-86 airplane (engine NK-86) in the cruising-flight mode at a height of 11 km we have r_a = 0.54 m, u_∞ = 236 m/sec, u_a = 389 m/sec, T_∞ = 216.7 K, T_a = 388 K, s = 1.648, h = 1.896 and Y_a = 0.0063. According to a strict numerical algorithm, at x_{10} = 54 m we have obtained ν_{10} = 2.26 m^2/sec, u_{10} = 31.3 m/sec, $H_{10} = 4.50 \cdot 10^4$ J/kg (T_{10} = 36.9 K), $Y_{10} = 1.29 \cdot 10^{-3}$, r_j = 1.93 m, and r_c = 1.815 m. We find the corresponding similarity parameters of the quasi-self-similar solution by linear approximation on the basis of the results of parametric calculations for the basic version (n = –0.2893, p = –0.8664, A = 1.334, and B = 1.546). Figure 2 gives the exact dependences of the temperature and the velocity on the axis of the jet behind the Il-86 airplane and the quasi-self-similar approximation. Transverse profiles correspond to the exact solution within the entire range of x with an error no higher than the error of the basic version.

2. In the second version, we approximate the exact solution by the quasi-self-similar one on the total length of the condensation trail from the cross section x_m of the maximum value $\langle Y_i \rangle$ = max of the average over the cross-section concentration of the condensate to the end cross section $x_{c.e}$. In the tail of the condensation trail, which is comparable to its total length, the concentration of the condensate is so low that this region virtually makes no contribution to the total mass of the condensate. Not to overstate the length of the trail in the model considered, we cut the condensation trail in the cross section where the concentration of the condensate on the axis amounts to 1% of its maximum. For the basic version we have obtained x_m = 48.7 m, ν_{10} = 3.94 m^2/sec, u_{10} = 54.7 m/sec, $H_{10} = 4.06 \cdot 10^4$ J/kg (T_{10} = 26.0 K), $Y_{10} = 1.27 \cdot 10^{-3}$, r_j = 2.30 m, r_c = 2.17 m, s = 1.712, h = 1.506, Y_a = 0.0039, and $Y_{s.i\infty} = 4.58 \cdot 10^{-5}$; in the end cross section $x_{c.e}$ = 1801 m, ν_{1e} = 1.509 m^2/sec, u_{1e} = 2.50 m/sec, H_{1e} = 1500 J/kg (T_{1e} = 1.25 K), $Y_{1e} = 5.81 \cdot 10^{-5}$, $r_{j.e}$ = 10.9 m, and $r_{c.e}$ = 2.30 m. The similarity parameters of quasi-self-similar prob-



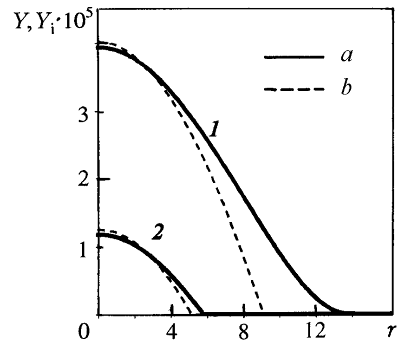


Fig. 3. Concentrations of the vapor Y (1) and ice Y_i (2) in the aerosol on the axis of the jet behind the Il-96 airplane with a deviation of +5 K in the atmospheric temperature: a) exact numerical solution; b) approximation on the basis of the quasi-self-similar solution constructed on the length of the condensation trail.

Fig. 4. Transverse distributions of the concentration of the vapor Y (1) and ice Y_i (2) in the aerosol at a distance of 3000 m in the jet behind the Il-96 airplane with a deviation of –5 K in the atmospheric temperature; a and b, notation as in Fig. 3.

lem (14) are $A = 1.5067$, $B = 2.0419$, $n = -0.2656$, and $p = m = g = -0.8542$. The solution is approximated by the polynomials of the second degree

$$F = 0.9911 - 0.2890\xi - 0.6133\xi^2\,, \quad W = Q = Z = 0.9881 - 0.9997\xi + 0.0757\xi^2\,. \tag{17}$$

The parametric calculations have shown that the dependences of the similarity parameters of the quasi-self-similar problem n, p, A, and B on the parameters s, h, Y_a, and $Y_{s.i\infty}$ of the initial problem (1)–(5) are more complex than in the first version. The dependences on the quantity $Y_{s.i\infty}/10^{-5}$ within the range from 2.46 to 9 can be approximated by polynomials of the second degree with coefficients of a_0 = –0.2223, –0.8425, 1.955, and 3.307; a_1 = –0.0133, –0.0014, –0.1250, and –0.3587; a_2 = 0.000853, –0.000205, 0.00589, and 0.01827 for n, p, A, and B, respectively.

As an example of the use of analytical formulas, we consider seasonal-latitudinal changes in the temperature in the cruising flight of an Il-96 aerobus.

With an increase of +5 K in the atmospheric temperature (T_∞ = 221.7 K) and constant propulsion we have [7] u_∞ = 238 m/sec, u_a = 410 m/sec, T_a = 292 K, s = 1.723, h = 1.503, Y_a = 0.00398, and $Y_{s.i\infty} = 8.96 \cdot 10^{-5}$. The numerical solution at x_m = 70 m gives ν_{10} = 3.92 m^2/sec, u_{10} = 37.7 m/sec, $H_{10} = 1.74 \cdot 10^4$ J/kg, T_{10} = 17.9 K, $Y_{10} = 8.73 \cdot 10^{-4}$, r_j = 2.81 m, and r_c = 2.34 m. The parameters s and h change by less than 0.7%, while the quantity Y_a changes by about 2%. At $Y_{s.i\infty} = 9 \cdot 10^{-5}$, the dependences of the similarity parameters n, p, A, and B on Y_a within the range 0.003–0.008 can be approximated by the polynomials of the second degree with coefficients of a_0 = –0.2801, –0.9262, 0.9150, and 0.5136; a_1 = 1.494, 17.75, 124.4, and 329.7; a_2 = 75.47, –1111, –6066, and –16,770 for n, p, A, and B, respectively. For the version under consideration we obtain $n = -0.2730$, $p = -0.8732$, $A = 1.314$, and $B = 1.560$. Figure 3 gives the relative concentrations of the vapor Y and ice Y_i on the jet axis as functions of the distance x.

Under winter conditions and high latitudes, deviations of –10, –20, and –30 K in ΔT_∞ are possible at a height of 11 km. For example, at –5 K (T_∞ = 211.7 K) we have [7] u_∞ = 235.2 m/sec, u_a = 397 m/sec, T_a = 280 K, s = 1.688, h = 1.502, Y_a = 0.0038, and $Y_{s.i\infty} = 2.46 \cdot 10^{-5}$. At x_m = 37 m, the numerical solution gives ν_{10} = 3.76 m^2/sec, u_{10} = 67 m/sec, $H_{10} = 5.32 \cdot 10^4$ J/kg, T_{10} = 33.5 K, $Y_{10} = 1.68 \cdot 10^{-3}$, r_j = 1.975 m, and r_c = 1.954 m. At $Y_{s.i\infty} = 2.5 \cdot 10^{-5}$, the dependences of the parameters n, p, A, and B on Y_a within the range 0.001–0.0045 can be approximated by the square polynomial: a_0 = –0.2823, –0.8952, 1.046, and 0.9186; a_1 = 7.866, 24.87, 273.5, and 645.7; a_2 = 104.6, –3180, –27,817, and –58,272 for n, p, A, and B, respectively. The calculated values of the similarity parameters are n

$= -0.2510$, $p = -0.8466$, $A = 1.684$, and $B = 2.531$. Figure 4 gives transverse distributions of the concentrations of the vapor Y and ice Y_i at the distance $x = 3000$ m.

Thus, we have suggested a description of the exhaust jet in the far field $x \geq 10^2 r_a$ using the quasi-self-similar solution in two versions of approximation of similarity parameters: (1) within the range $\Delta x = (00^2 – 10^3) r_a$; (2) on the length of the condensation trail $\Delta x = x_m - x_{c.e}$. Analytical formulas allow calculation of the initial conditions for the problem of interaction between the jet and the end vortices, estimation of the initial parameters of the condensation trail, and calculation of the total and average characteristics of the condensation trail.

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NOTATION

t, time, sec; x and r, longitudinal and transverse coordinates, m; u and v, velocity components, m/sec; ρ, density, kg/m^3; T, temperature, K; H, gas enthalpy, J/kg; Y, relative mass concentration of the vapor; μ, coefficient of turbulent dynamic viscosity, kg/(m·sec); k, coefficient of thermal conductivity, W/(m·sec); D, coefficient of diffusion, m^2/sec; $\nu = \mu/\rho$, coefficient of turbulent kinematic viscosity, m^2/sec; L, latent heat of evaporation (condensation), J/kg; R, universal gas constant, J/(kmole·K); m and m_w, molar masses of air and water, kg/kmole; similarity parameters: concurrency $1/s = u_\infty/u_a$, parameter of heating $h = H_a/H_\infty$, relative mass concentration of the vapor at the nozzle cut Y_a and in the atmosphere Y_∞, concentration of the saturated vapor above ice $Y_{s.i\infty}$; $\mathrm{Re} = u_\infty r_a/\nu_a$, Reynolds number; $\mathrm{Pr} = C_p\mu/k$, Prandtl number; $\mathrm{Sc} = \mu/\rho D$, Schmidt number; $f(\eta)$, $w(\eta)$, $q(\eta)$, $z(\eta)$, $\eta = r^2/8t^{n+1}$, ξ, n, g, m, and p, functions and parameters of the self-similar solution. Subscripts: a, average, averaged; ∞, parameters at the cut of the nozzle of radius r_a and in the atmosphere; c, condensation trail; e, end; i, ice; j, jet; m, maximum; p, pressure; s, saturation; w, water; 0, initial; 1, principal terms of the disturbances of the sought functions.

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